How people interpret an uncertain *if* \(^1\)

Andy Fugard, Niki Pfeifer, Bastian Mayerhofer, and Gernot Kleiter

Department of Psychology
University of Salzburg

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Consider a fair die with the following patterns on the sides:

- Square
- Circle
- Square
- Square
- Circle
- Circle

The die is thrown and lands with a side facing up.
Consider a fair die with the following patterns on the sides:

![Die Faces]

The die is thrown and lands with a side facing up.

What is the probability that:

if the side shows a square, then the side shows red?
Conjunction interpretation of *if*

if the side shows a square, then the side shows red

\[ |\text{square} \land \text{red}| = 2 \]

\[ |\text{sides}| = 6 \]

\[ P(\text{square} \land \text{red}) = \frac{2}{6} \]
Material conditional interpretation of if

\[ P(\text{square} \Rightarrow \text{red}) = \frac{2 + 3}{6} \]

if the side shows a square, then the side shows red

\[ |\text{square} \land \text{red}| = 2 \]
\[ |\neg \text{square}| = 3 \]
\[ |\text{sides}| = 6 \]
Conditional event interpretation of *if*

*if the side shows a square, then the side shows red*

\[ |square \land red| = 2 \]
\[ |square| = 3 \]
\[ P(red|square) = \frac{2}{3} \]
if the side shows a square, then the side shows red

\[
|\text{square} \land \text{red}| = 2 \\
|\text{square}| = 3 \\
P(\text{red} | \text{square}) = \frac{2}{3}
\]
What’s to come

1. Speedy overview of probabilistic truth-table tasks
2. More detail on the development of our task
3. Modal responses
4. *Intra*-individual differences
5. The joys of response times
Typically about cards drawn randomly from a shuffled deck
Each card has an element from two dimensions
Joint frequencies are provided numerically, e.g.,

1  yellow circle
4  yellow diamonds
16 red circles
16 red diamonds

Task: what is the probability that

*If the card is yellow then it has a circle printed on it?*
Conjunction: a partial representation/computation

- Johnson-Laird and Byrne (2002): implicit mental model
- Evans et al. (2003): incomplete execution of the Ramsey test
Explanations for responses

Conjunction: a partial representation/computation
- Johnson-Laird and Byrne (2002): implicit mental model
- Evans et al. (2003): incomplete execution of the Ramsey test

Conjunction: a ‘permissible’ interpretation
Edgington (2003): presence of word ‘true’ triggers process:
1. Consider when ‘if $A$, then $B$’ gets truth value $true$
2. What’s the probability of this? $P(A \land B)$
Conjunction: a partial representation/computation

- Johnson-Laird and Byrne (2002): implicit mental model
- Evans et al. (2003): incomplete execution of the Ramsey test

Conjunction: a ‘permissible’ interpretation

Edgington (2003): presence of word ‘true’ triggers process:

1. Consider when ‘if A, then B’ gets truth value true
2. What’s the probability of this? $P(A \land B)$

Conditional event: a standardized conjunction

- Complete Ramsey test (Evans et al., 2003)
- Mental models theory also characterizes the data
- Related to a three-valued logic (considered by Wason, 1966)
- Three-valued conditional and probability semantics united by the coherence approach (Coletti & Scozzafava, 2002)
Old idea: to use a logic you must first map the inference task to the formalism

Newer idea: use logics (plural) and their parameters to model interpretation in human reasoning

**Interpretation:** parameter setting

**Derivation:** inference once parameters set
Old idea: to use a logic you must first map the inference task to the formalism

Newer idea: use logics (plural) and their parameters to model interpretation in human reasoning

**Interpretation:** parameter setting

**Derivation:** inference once parameters set

Correctness with respect to interpretation

But how do people reason about interpretations?
Our task

- **Graphical**, not numerical, depiction without biasing towards the joints
- Frequencies obtained by **counting**
- **Systematic enumeration** of all 84 joint frequencies: 
  \((A \land B, A \land \neg B, \neg A \land B, \neg A \land \neg B)\)
- So more items than typically used
- Computer controlled so we get **response times**
- Responses of form ‘x out of y’ rather than probabilities
Die sides represented as a contingency table

<table>
<thead>
<tr>
<th></th>
<th>square</th>
<th>circle</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>red</strong></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>blue</strong></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Die sides represented as a contingency table

(Order randomized on the sides.)

<table>
<thead>
<tr>
<th></th>
<th>square</th>
<th>circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>blue</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Enumerating the die sides

- How many ways can make four numbers sum to 6? 
  $(0 + 0 + 0 + 6, 0 + 0 + 1 + 5, \ldots)$
- How many ways can the numbers be plugged into a contingency table?

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_3$</td>
<td>$p_4$</td>
<td>$p_5$</td>
<td>$p_6$</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>$4! / 3! = 4$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>$4! / 2! = 12$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>$4! / 2! = 12$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>$4! / (2!2!) = 6$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>$4! / 2! = 12$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>$4! = 24$</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>$4! / 3! = 4$</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>$4! / 3! = 4$</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>$4! / (2!2!) = 6$</td>
</tr>
</tbody>
</table>

$\sum = 84$
Enumerating the die sides

- How many ways can make four numbers sum to 6? 
  \((0 + 0 + 0 + 6, 0 + 0 + 1 + 5, \ldots)\)
- How many ways can the numbers be plugged into a contingency table?

<table>
<thead>
<tr>
<th>Partition</th>
<th>(p_1)</th>
<th>(p_2)</th>
<th>(p_3)</th>
<th>(p_4)</th>
<th>Permutations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>(4!/3!) = 4</td>
</tr>
<tr>
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<td>0</td>
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<td>0</td>
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</tr>
<tr>
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<td>0</td>
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<td>1</td>
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<tr>
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<td>1</td>
<td>3</td>
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</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>(4!/(2!2!)) = 6</td>
</tr>
</tbody>
</table>

\[\sum = 84\]
Experiment 1

- Presented in a lecture theater
- Computer controlled, presented by data projector
- 10 seconds per-item
- 66 Psychology students (57 females and 9 males)
- Aged 20–40 ($M = 23.8; SD = 3.5$)

Experiment 2

- Individual testing, paid
- Self-paced
- Computer controlled, with a custom response box
- Response times collected
- 65 students (32 females and 33 males)
- Aged 18–30 ($M = 22.9; SD = 2.9$)
- No psychologists or people trained in logic
## Experiments

### Experiment 1
- Presented in a lecture theater
- Computer controlled, presented by data projector
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- 66 Psychology students (57 females and 9 males)
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### Experiment 2
- Individual testing, paid
- Self-paced
- Computer controlled, with a custom response box
- Response times collected
- 65 students (32 females and 33 males)
- Aged 18–30 ($M = 22.9; SD = 2.9$)
- No psychologists or people trained in logic
Hier ist der Würfel 1
Seine Seiten sehen so aus:

Der Würfel wird geworfen. Eine zufällige Seite zeigt nach oben. Wie sicher können Sie sein, dass der folgende Satz stimmt?
Wenn die Seite ein Viereck zeigt, dann zeigt die Seite rot.

Wenn die Seite ein Viereck zeigt, dann zeigt die Seite rot.
Here is die 1

Its sides look like this:

The die is thrown. A random side shows up. How sure can you be that the following statement is true?

If the side shows a square, then the side shows red.

out of
Der Würfel wird geworfen. Eine zufällige Seite zeigt nach oben. Wie sicher können Sie sein, dass der folgende Satz stimmt?

Wenn die Seite ROT zeigt, dann zeigt sie ein VIERECK.
How they responded: individual testing
Response classifications

- Responses classified as:
  - $B | A$
  - $A | B$
  - $A \Rightarrow B$
  - $B \Rightarrow A$
  - $A \land B$
  - Other

- A priori we know that not all items distinguish interpretations
- Not always a problem (e.g., if a confusable response is never actually used)
- Empirically we find the ‘good’ items: 46 for both experiments
Classifications of participants by their modal response

Response class

- B|A
- A => B
- A & B
- A|B
- B => A
- Other

Pilot (N = 18)
Experiment 1 (N = 66)
Experiment 2 (N = 65)

Percent of Participants

- B|A
- A => B
- A & B
- A|B
- B => A
- Other

Graph showing the distribution of responses for each category.
How do our results compare?

Response class

Percent of Participants

B|A  A & B  Other

Evans et al., 2003
Oberauer & Willhelm, 2003
Oberauer et al., 2007
Our data (mean of 3 experiments)
Experiment 1

(a) B|A

(b) A => B

(c) A and B

Experiment 2

(d) B|A

(e) A => B

(f) A and B
**Experiment 1**

(a) $B|A$

![Graph](image)

$r = .82$

(b) $A \Rightarrow B$

![Graph](image)

(c) $A$ and $B$

![Graph](image)

**Experiment 2**

(d) $B|A$

![Graph](image)

$r = .68$

(e) $A \Rightarrow B$

![Graph](image)

(f) $A$ and $B$

![Graph](image)

$\text{Experiment 1}$

$\text{Experiment 2}$
**Experiment 1**

(a) $B | A$

Proportion of participants

Item position

$r = .82$

(b) $A \Rightarrow B$

Item position

(c) $A$ and $B$

$\ r = -.73$

**Experiment 2**

(d) $B | A$

$r = .68$

(e) $A \Rightarrow B$

(f) $A$ and $B$

$r = -.73$
What happens within-participant?
|     | B\(|A| | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
|-----|-------|---|---|---|---|---|---|---|---|---|---|
| A\(\land\)B | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| A\(\Rightarrow\)B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
Example: finding split point

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>A</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A∧B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>A⇒B</td>
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<td>0</td>
</tr>
<tr>
<td>Other</td>
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<td>0</td>
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</table>

0 5/9
Example: finding split point

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<th>1</th>
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<th>1</th>
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</tr>
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<tbody>
<tr>
<td>B</td>
<td>A</td>
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<td>0</td>
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<td>1</td>
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<tr>
<td>A∧B</td>
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<td>Other</td>
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</table>

0/1 5/8
Example: finding split point

<table>
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<tr>
<th></th>
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<th>1</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$B</td>
<td>A$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A \land B$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>$A \Rightarrow B$</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

$0/2$  $5/7$
Example: finding split point

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>A</th>
<th>A∧B</th>
<th>A⇒B</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>0</td>
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<td>0</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>5/6</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

0/3  5/6
Example: finding split point

<table>
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<tr>
<th></th>
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<th>0</th>
<th>0</th>
<th>0</th>
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<th>1</th>
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</thead>
<tbody>
<tr>
<td>$B</td>
<td>A$</td>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>Other</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>

0/4  5/5
Example: finding split point

<table>
<thead>
<tr>
<th>B</th>
<th>A</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
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<tbody>
<tr>
<td>A∧B</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>A⇒B</td>
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</tbody>
</table>

**Good bet for switch**
Example: finding split point

<table>
<thead>
<tr>
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1/5 4/4
Example: finding split point

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2/6 3/3
Example: finding split point

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3/7 2/2
Non-uniquely classifiable responses weighted down

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</table>
Where did switchers come from? (Modal response to left)

(from 36 out of 65 who switch to conditional event)
Properties of split points

(a) Split points

(b) Modal response left

(c) B|A response right
P37, shift from $A \land B$ to $B|A$

‘In the beginning [I] always [responded] ‘out of 6’, but then somewhere in the middle... Ah! It clicked and I got it. I was angry with myself that I was so stupid before.’
Experiment 1: we suspected that people’s computations were interrupted by the 10 second limit
Conjunction resulted from being cut-off mid conditional-event computation
Task adaptation allowed the conditional event to be computed
- Experiment 1: we suspected that people’s computations were interrupted by the 10 second limit
- Conjunction resulted from being cut-off mid conditional-event computation
- Task adaptation allowed the conditional event to be computed
- BUT learning still occurred for self-paced Experiment 2
Explanations

**Insight effect?**
- Sudden shift of strategy
- Some spontaneous ‘Aha!’ reports

But... no impasse
Normally goal difficult for insight problems; here the interpretation shifts and the goal is always easy to achieve

Why?
Clue from Politzer (1981): learning on a verity logic TT task
Going through cases seems to change interpretation
The antecedent frequencies become salient, e.g., shapes and colors are easier to separate
Insight effect?

- Sudden shift of strategy
- Some spontaneous ‘Aha!’ reports
- But... no impasse
Explanations

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- Sudden shift of strategy
- Some spontaneous ‘Aha!’ reports
- But... no impasse
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Explanations

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**Why?**

- Clue from Politzer (1981): learning on a verity logic TT task
- Going through cases seems to change interpretation
- The antecedent frequencies become salient, e.g., shapes and colors are easier to separate
What about RTs?

Consider the computations required:

\[
\begin{align*}
|A \land B| &= f_1 \\
|A| &= f_2 \\
\frac{|A|}{P(B|A)} &= \frac{f_1}{f_2}
\end{align*}
\]

\[
\begin{align*}
|A \land B| &= f_1 \\
|Sides| &= 6 \\
\frac{|Sides|}{P(A \land B)} &= \frac{f_1}{6}
\end{align*}
\]
What about RTs?

Consider the computations required:

\[
|A \land B| = f_1 \\
|A| = f_2 \\
\frac{|A \land B|}{|A|} = f_1 / f_2 \\
\]

\[
P(B|A) = f_1 / f_2 \\
\]

\[
|A \land B| = f_1 \\
|Sides| = 6 \\
\frac{|Sides|}{P(A \land B)} = f_1 / 6 \\
\]

Hypothesis: conjunction faster as denominator constant
Overall speed-up

![Graph showing overall speed-up with RT (ms) on the y-axis and Item position on the x-axis. The data points are scattered, and a trend line is visible, indicating a decrease in RT as Item position increases.]
Conjunction faster

(ΔAIC = −26, LLR $\chi^2(5) = 35.9, p < 0.0001$)
P34, persistent \( A \land B \)

‘I only looked at the shape and the color, and then always out of 6; this was the quickest way.’
Conclusions

- Conditional event interpretation most common
- People converge on a conditional event interpretation
- Evidence of insight effect
- Studying trajectories of interpretation reveals reasoning about interpretation
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- Conditional event interpretation most common
- People converge on a conditional event interpretation
- Evidence of insight effect
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Thank you!
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- Thanks to Leonhard Kratzer and David Over for comments
- Data reported in this talk are described by Fugard, Pfeifer, Mayerhofer, and Kleiter (2009, to be presented at WUPES)


