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Probabilistic theories of reasoning need pragmatics too: Modulating relevance in uncertain conditionals

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ABSTRACT

According to probabilistic theories of reasoning in psychology, people's degree of belief in an indicative conditional 'if *A*, then *B*' is given by the conditional probability, $P(B \mid A)$. The role of language pragmatics is relatively unexplored in the new probabilistic paradigm. We investigated how consequent relevance affects participants' degrees of belief in conditionals about a randomly chosen card. The set of events referred to by the consequent was either a strict *superset* or a strict *subset* of the set of events referred to by the antecedent. We manipulated whether the superset was expressed using a disjunction or a hypernym. We also manipulated the source of the dependency, whether in long-term memory or in the stimulus. For subset-consequent conditionals, patterns of responses were mostly conditional probability followed by conjunction. For superset-consequent conditionals, conditional probability responses were most common for hypernym dependencies and least common for disjunction dependencies, which were replaced with responses indicating inferred consequent irrelevance. Conditional probability responses were also more common for knowledge-based than stimulus-based dependencies. We suggest extensions to probabilistic theories of reasoning to account for the results.

1. Introduction

Consider the following conditional statement (based on an imperative conditional by Ross, 1944:38):

If Hans slipped the letter into the letter box (*A*), then he slipped the letter into the letter box (*A*) or he burned the letter (*B*).

In classical logic this may be formalized using the material conditional ($\supset$) and inclusive disjunction ($\vee$) as $A \supset A \vee B$, which is a tautology, i.e., true irrespective of the particular *A* and *B* filled in. However, many participants think that conditionals of the form 'if *A*, then *A* or *B*' are false. As a consequence, the rule allowing this inference to be drawn is absent from the mental logic theory of reasoning (Braine et al., 1998) and has the lowest availability in the ANDS mental rules theory (Rips, 1983:62). More recent analyzes of the semantics of the natural language 'or' do not have disjunction introduction as a valid inference rule (Geurts, 2005). From a Gricean perspective, asserting a disjunction when one of the disjuncts is known to be true violates the Maxim of Relation (be relevant!), of Quantity (be informative!), and of Manner (avoid obscurity!) (Verhoeven, 2007).

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$$\text{(a)} \ \frac{\neg A}{A \supset B} \qquad \text{(b)} \ \frac{\neg A}{\neg A \vee B} \qquad \text{(c)} \ \frac{B}{A \supset B} \qquad \text{(d)} \ \frac{B}{\neg A \vee B}$$

Fig. 1. Classical logic rules introducing implications in the paradoxical manner (a and c) and their corresponding disjunctive-introduction relatives (b and d), showing clearly how an irrelevant term is introduced.

Table 1
Semantic values of the material conditional ($A \supset B$), conjunction ($A \wedge B$), and the conditional event ($B \mid A$).

A	B	$A \supset B$	$A \wedge B$	$B \mid A$
True	True	True	True	True
True	False	False	False	False
False	True	True	False	Void
False	False	True	False	Void

The importance of interpretation in reasoning, driven by a range of contextual factors including those concerning pragmatics, has long been emphasized (e.g., Henle, 1962; Smedslund, 1970; Adler, 1984; Politzer, 1986; Stenning and van Lambalgen, 2008). Before participants can reason about natural language sentences, they must first interpret the sentences, and the task they have been asked to perform. It seems unlikely that people interpret conditionals of the form 'if A, then A or B' as $A \supset A \vee B$, or at least if they do, it is unlikely they view the task as only tautology checking.

Recently conditional probability interpretations of the natural language conditional have become popular in psychology (see, e.g., Over and Evans, 2003; Oaksford and Chater, 2007; Pfeifer and Kleiter, 2009), though the semantics has a longer history in philosophy (see, e.g., Adams, 1975; Edgington, 1995; Bennett, 2003). According to these accounts, participants' degree of belief in an indicative conditional 'if A, then B' is given by the conditional probability $P(B \mid A)$. This formalisation solves many problems with the material conditional interpretation of the if-then, for instance the so-called paradoxes of the material conditional interpretation disappear (discussed in the psychology of reasoning literature by Evans et al., 2005). Consider the following conditional:

If the popular video game Pac-Man is loosely based on Dostoyevsky's Crime and Punishment, then politicians are honest.[1]

According to the material conditional interpretation, since the antecedent is false (from an implicit premise), the whole conditional is true irrespective of the truth of its consequent. Another paradox is illustrated by this example:

If politicians are honest, then Mozart was born in Salzburg.

Since the consequent is true (again from an implicit premise), the whole conditional is true irrespective of the truth of the antecedent and its relationship with the consequent. (See Fig. 1(a and c) for valid classical logic inference rules which formalize the paradoxes.) According to the conditional probability account of conditionals, however, sentences of both kinds are probabilistically uninformative, matching participants' inferences (Pfeifer and Kleiter, 2006, in press).

The traditional approach to probability defines conditional probabilities, $P(B \mid A)$, in terms of unconditional probabilities using the ratio formula:

$$P(B|A) \stackrel{\text{def}}{=} \frac{P(A \wedge B)}{P(A)}, \quad \text{if } P(A) > 0$$

An alternative approach introduced by de Finetti (1937/1980) is to define a logic where the conditional event, $B \mid A$, also exists as a (non-classical) logical connective and is not only defined in terms of unconditional probabilities. The semantic values of $B \mid A$ are the same as those for conjunction and material conditional when A is true and *void* when A is false. (See Table 1.) Probability functions can be applied directly to these conditional events.

The values for the conditional event correspond to the pattern of responses sometimes known in psychology as the 'defective truth table' (Wason, 1966; Johnson-Laird and Tagart, 1969). Evidence that this pattern of responses is due to a conditional event interpretation is found in empirical results showing that those who give a conditional probability interpretation of uncertain conditionals also produce the de Finetti table for certain conditionals (Evans et al., 2007). Politzer et al. (2010) found evidence that people treat conditional bets, 'I bet you if A then B', similarly to indicative conditionals, with *win* corresponding with *true*, *lose* corresponding with *false*, and *void* when the bet is called off. This is again consistent with a proposal by de Finetti (1937/1980).

The conditional event interpretation of the natural language conditional also allows nonmonotonic reasoning (e.g., Gilio, 2002), making it possible for inferences to be withdrawn as more information becomes available, which is a common feature of human inference (Politzer and Bourmaud, 2002; Stenning and van Lambalgen, 2005, 2008; Pfeifer and Kleiter, 2005, 2009).

[1] False antecedent thanks to Twitter #NotAFact.

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Now reconsider the sentence introduced earlier:

(1) If Hans slipped the letter into the letter box (*A*), then he slipped the letter into the letter box (*A*) or he burned the letter (*B*).

When the conditional is formalized as a conditional event, its probability is $P(A \vee B \mid A) = 1$. In an exploratory study run by us with 20 participants, involving inferences about a thrown six-sided die, 55% of participants inferred that the probability of 'if the die shows a 5, then it shows a 5 or a 6' is 0 and only 15% responded that the probability is 1, consistent with the conditional event interpretation. Similarly to the case of classical logic, the direct translation into the language of probability logic does not match participants' inferences. The material conditional collapses to disjunction ($A \supset B \equiv \neg A \vee B$) so there is a close relationship between the paradoxes of the material conditional and disjunction-introduction, as illustrated by Fig. 1, which in turn is closely related to sentences of the form $A \supset A \vee B$.

Consider now the following conditional statement:

(2) If Hans is drinking coffee (*A*), then he is drinking a liquid (*B*).

Sentences (1) and (2) both have in common that the set of events referred to by their consequents is a superset of the set of events referred to by their antecedents. The difference is that in sentence (1), the superset is expressed using a disjunction, and in sentence (2) the superset is expressed using a hypernym of the term in the antecedent. For a related item in our exploratory study, 'if the die shows a 2, then it shows an even number', the probability 1 was inferred by almost all participants. This is consistent with the conditional event interpretation of the conditional.

The main goal of the present study is to extend these exploratory results and systematically investigate the effects of consequent relevance, using a task derived from probabilistic truth table tasks (Evans et al., 2003; Oberauer and Wilhelm, 2003; Gauffroy and Barrouillet, 2009; Fugard et al., in press). We manipulated whether the superset was expressed using a disjunction or a hypernym. We also manipulated the source of the dependency, whether in long-term memory or in the stimulus. Investigations of the source of dependencies have a long history in psychology. For instance people are more likely to believe that an argument is valid if the conclusion is compatible with their beliefs than if it is incompatible with their beliefs, irrespective of the logical validity of the argument (Evans et al., 1983).

Participants were shown various collections of four cards with patterns and numbers printed only on one side (see Fig. 2 for an example). They were asked to imagine that the cards were shuffled, placed face down, and a random card selected. The task was then to infer how sure they were that a conditional holds of such a randomly chosen card. The relationship between antecedent and consequent was expressed in four ways to test the effect this had on interpretation. For superset-consequent conditionals: (i) using disjunction interpretable using only knowledge (e.g., 'if it shows a square, then it shows a square or a circle'); (ii) using a familiar hypernym (e.g., 'if it shows a square, then it shows a shape'); (iii) representing the relation in the stimulus by the patterns of shapes and numbers (e.g., 'if it shows a 1, then it shows a shape'); and (iv) combining a stimulus-based presentation with a disjunction (e.g., 'if it shows a 1, then it shows a square or a circle'). A well replicated effect in probabilistic truth table tasks is that most participants give a conditional event interpretation and the remainder a conjunction interpretation. We aimed to discover the effect of the different expressions of the superset on the frequencies of these interpretations. For the example in Fig. 2, according to the conditional event interpretation the probability is 1 and conjunction interpretation is 1/4 for all four ways to express the relationship. We expected people to respond with 0 if they thought the conclusion was irrelevant.

When asked about the truth value of the classical logic statement $A \vee B \supset B$, the correct answer is 'can't tell': in some models where $A \vee B$ is true, *B* will be true and in others it will be false. In the probabilistic framework it is possible to infer how likely the statement is. For instance consider a randomly chosen card, as per the task outlined above, and the conditional,

If the card shows a 1 or a 2, then the card shows a 1.

When interpreted as a conditional event, the probability is $P(x = 1 \mid x = 1 \vee x = 2) = 1/2$. The present study also investigates whether how the relations are expressed influences the inferences drawn for subset-consequent conditionals. We predict these inferences would be relevant, leading to the conditional event interpretation, as the antecedent adds information not present in the consequent.

Previously we found that when participants solve a series of trials on a graphical probabilistic truth table task, some participants spontaneously shift interpretation from conjunction to conditional event (Fugard et al., in press). For this task, the logical form of the conditional remained fixed. In the present study we also investigate whether people shift interpretations when the logical form of the conditional changes between trials.

2. Experiment

2.1. Method

2.1.1. Participants

There were 64 participants (32 females and 32 males), whose ages ranged from 18 to 45 ($M = 24.1$; $SD = 4.9$). All but one were students. Students of psychology, mathematics, or with a special background in formal logic, were not included in the sample. We paid 5 Euros for participation.

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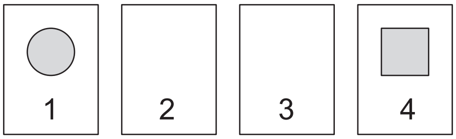

Fig. 2. Example cards shown to participants.

2.1.2. Materials and procedure

The data were collected as part of a larger study of reasoning. The task was presented to participants in a booklet, beginning with the following introduction:

> Hans has four cards. One card shows a One, one card shows a Two, one card shows a Three, one card shows a Four. So, two of the cards show even numbers and two cards show odd numbers. Additionally, one of the cards shows a square and another shows a circle. So two cards show a shape and two show no shape. The cards are different for each task. The cards are only printed on the front side!

Each item was printed on a single page. On each page, participants were shown a graphical representation of the cards (for instance as in Fig. 2), underneath which was the text:

> Hans takes the four cards, shuffles them well and puts them face down in front of him on the table. Then he draws a random card and puts it, again face down, on the table. Hans cannot see the front side of the cards.

They were then given a conditional of the form 'If the card shows a —, then the card shows a —' (*Wenn die Karte ein(e) — zeigt, dann zeigt die Karte ein(e) —*), with the placeholders filled as explained below, and were asked: 'How sure can Hans be that the following statement holds?' (*Wie sicher kann Hans sein, dass die folgende Behauptung stimmt?*). Previous studies have asked about the truth of the conditional, which it has been argued could bias participants towards a conjunction interpretation of the conditional (Edgington, 2003). Using the word 'holds' avoids this problem. Responses were made on a five-point rating scale from 0 (absolutely sure that the statement does not hold) to 1 (absolutely sure that the statement holds), in quarter steps (0, 1/4, 1/2, 3/4, 1). There were two warm-up trials where participants were asked to assess the probability of an atomic event (e.g., 'the card shows a 2'), and eight trials testing inferences about conditionals. Table 2 shows the items used in the experiment. For instance item 5 in the numerical antecedent phrasing was 'If the card shows a two, then the card shows an even number' (*Wenn die Karte eine Zwei zeigt, dann zeigt die Karte eine gerade Zahl* in the German original).

We used a fully crossed mixed design. Within-participant we varied: the set relationship between antecedent and consequent (subset- versus superset-consequent); how the set relation was expressed (hypernym versus disjunction); and whether knowledge was sufficient or the stimulus was also required to understand the relationship between antecedent and consequent. Between-participant we crossed how the antecedent was expressed (using numerals or shapes), the item order (one randomization: forwards and reversed), and sex.

2.2. Results

Statistical analyzes were performed using R version 2.9.1 (R Development Core Team, 2009). Generalized linear mixed-effects models (GLMMs) were fitted using the lme4 package (Bates, 2008; Baayen et al., 2008).

2.2.1. Effect of set relation formulation

Fig. 3(a) shows the proportion of conditional event (or material conditional—the two are indistinguishable for these items) responses for superset-consequent conditionals. The competence model prediction for these items is a probability of 1. Binomial GLMMs with a logit link were fitted predicting the probability of a conditional event response and allowing the intercept to vary by participant. Conditionals expressed without a disjunction were more likely to be given a conditional event

Table 2
Items presented to participants. Items were presented as if-then (in German as *wenn-dann*) sentences. Here → is used to abbreviate the natural language if-then and 'the card shows' is also omitted (see text for complete examples).

	Conditional	
	Numeral antecedent	Shape antecedent
1.	Even → 2	Shape → circle
2.	4 → shape	Square → even
3.	1 → 1 or 3	Circle → circle or square
4.	3 → square or circle	Circle → 1 or 3
5.	2 → even	Square → shape
6.	2 or 4 → square	Square or circle → 2
7.	1 or 3 → 1	Circle or square → circle
8.	Odd → circle	Shape → 1

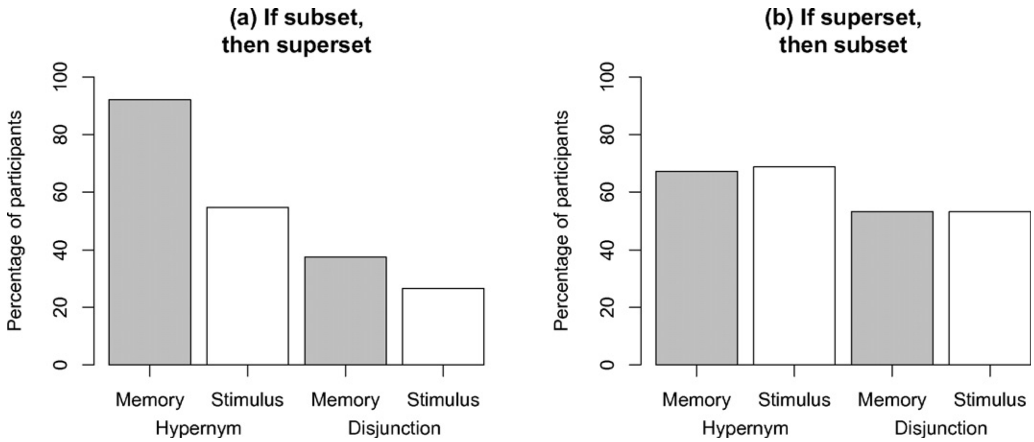

Fig. 3. Percentage of participants ($N = 64$) responding with conditional event responses, (a) for superset-consequent conditionals, and (b) for subset-consequent conditionals, as a function of how the relationship was expressed.

Table 3
Distributions of responses in percentage of participants ($N = 64$) for the eight items.

Interpretation	Response	Disjunction		Hypernym	
		Mem.	Stim.	Mem.	Stim.
	(a) If subset, then superset				
$B \mid A$ or $A \supset B$	1	38	27	92	55
$A \wedge B$	1/4	5	13	3	22
Zero	0	45	20	2	5
$A \mid B$	1/2	11	39	0	16
Other	3/4	2	2	2	2
	(b) If superset, then subset				
$B \mid A$	1/2	53	53	67	69
$A \wedge B$	1/4	28	8	23	20
$A \supset B$	3/4	0	4	0	2
Zero	0	17	27	8	3
$A \mid B$	1	2	8	2	6

Note" Mem. = long-term memory; Stim. = stimulus.

response than were those expressed with a disjunction ($z = 7.8$, $p < 0.001$). Also for disjunctively expressed and non-disjunctively expressed supersets, conditional event responses were less common when the relation was found in the stimulus, rather than in long-term memory ($z = -4.9$, $p < 0.001$). There was also an interaction between whether the dependency was expressed in stimulus and whether a disjunction was used ($z = -3.0$, $p = 0.002$): the difference between long-term memory and stimulus was greater for the hypernym condition compared to the disjunction condition.

Table 3(a) breaks down the responses in more detail. Zero responses are most common in the presence of disjunctions, and especially common for the long-term memory (versus stimulus) condition. Evidence was found for what is equivalent to a reversed conditional event, $A \mid B$, for the stimulus-disjunction conditional. The answer giving this interpretation is 1/2, which might also indicate 'don't know'. (See the discussion for more on this possibility.)

Fig. 3(b) shows the proportion of conditional event responses for subset-consequent conditionals. The conditional event response was always 1/2. Here no difference was found between long-term memory and stimulus dependent expressions of the dependency ($z = -0.3$, $p = 0.76$). Relations expressed as a hypernym resulted in more conditional event responses than did disjunctively expressed relations ($z = 3.1$, $p = 0.002$). There was no interaction found between whether the dependency was expressed in stimulus and whether a disjunction was used ($z = -0.33$, $p = 0.74$).

Table 3(b) breaks down the responses in more detail. One effect worth highlighting is that conjunction responses are least common for the stimulus-disjunction condition, where they appear to be replaced by zero responses. Conjunction responses were most common for the memory-disjunction condition.

2.2.2. Shifts of interpretation

Binomial GLMMs were fitted allowing the intercept to vary by participant and by item. The probability of a conditional event response increased ($z = 2.00$, $p = 0.045$) and the probability of a conjunction response decreased ($z = -2.55$, $p = 0.01$) as a function of item position, replicating previous results for stimulus-dependent conditionals without embedded disjunctions (Fugard et al., in press).

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Table 4
Correlations (Spearman's ρ) between counts of interpretations of each type by condition.

		1	2	3	4	5	6	7	8	9	10	11
	(a) If superset, then subset											
	Disjunction											
1.	$B\mid A$											
2.	$A \wedge B$	-0.62^{**}										
3.	Zero	-0.58^{**}	-0.04									
	Hypernym											
4.	$B\mid A$	0.36^{**}	-0.42^{**}	-0.05								
5.	$A \wedge B$	-0.30^{*}	0.47^{**}	0.01	-0.76^{**}							
6.	Zero	-0.34^{**}	0.12	0.14	-0.38^{**}	-0.14						
	(b) If subset, then superset											
	Disjunction											
7.	$B\mid A$	0.14	-0.17	-0.11	0.19	-0.35^{**}	0.18					
8.	$A \wedge B$	-0.30^{*}	0.45^{**}	0.04	-0.23	0.37^{**}	-0.13	-0.16				
9.	Zero	-0.11	-0.17	0.36^{**}	0.01	0.05	-0.07	-0.52^{**}	-0.20			
	Hypernym											
10.	$B\mid A$	0.20	-0.45^{**}	0.13	0.46^{**}	-0.55^{**}	-0.05	0.44^{**}	-0.50^{**}	-0.01		
11.	$A \wedge B$	-0.18	0.32^{**}	0.12	-0.41^{**}	0.63^{**}	-0.17	-0.42^{**}	0.58^{**}	0.06	-0.60^{**}	
12.	Zero	-0.01	0.01	-0.02	-0.10	-0.16	0.42^{**}	0.03	-0.09	0.23	-0.27^{*}	-0.12

* $p < 0.05$.
** $p < 0.01$.

2.2.3. Relationships between interpretations

Twelve sumscores were computed representing how often participants gave a conditional event, conjunction, or zero response to an item, grouped by whether the superset was in the antecedent or consequent and whether it was expressed using a hypernym or using disjunction. The range of each score was 0–2. Table 4 shows correlations between the scores.

All positive correlations were between responses of the same type, e.g., conditional event. All six correlations between the conjunction response scores were statistically significant. Only 3 of the conditional event response scores were statistically significant: a conditional event interpretation on the subset-consequent conditionals usually did not correlate with a conditional event interpretation on the superset-consequent conditionals. The exception was for the interpretation of relations expressed using a hypernym, which did carry between the conditions.

Finally, how does irrelevance aversion on superset-consequent problems relate to interpretation of subset-consequent problems? There was only one statistically significant correlation, and this just predicted a zero response. Thus it appears that whatever trait influences conditional event versus conjunction interpretations is independent of inferences about consequent relevance, i.e., we found no evidence that conditional event responders and conjunction responders are differentially affected by issues of relevance.

3. Discussion

We investigated inferences about consequent relevance, one aspect of pragmatics, in tasks asking participants to infer their degree of belief in uncertain conditionals. For subset-consequent conditionals, patterns of responses were mostly conditional probability followed by conjunction responses, with few material conditional responses, replicating a well known effect in probabilistic truth table tasks (Evans et al., 2003; Oberauer and Wilhelm, 2003; Gauffroy and Barrouillet, 2009). Conditional event responses were slightly less common for supersets expressed using disjunction than for supersets expressed using a hypernym.

For superset-consequent conditionals, conditional probability responses were most common for hypernym dependencies (e.g., 'If the card shows a 2, then the card shows an even number') and least common for disjunction dependencies (e.g., 'If the card shows a 2, then the card shows a 2 or a 4'), which were replaced with responses indicating inferred consequent irrelevance.

If we restrict to the domain of the four cards and interpret the conditional in the language of probability logic, then

$$P(x = 2 \vee x = 4 | x = 2) = P(even(x) | x = 2),$$

however most people's degree of belief in the former type of conditional was 0 whereas that of the latter type was 1. One explanation for this is that the disjunctive but not the hypernym formulation violates the Gricean maxim of Quantity (Grice, 1989). Given that we know $x = 2$, weakening this to a disjunction including $x = 2$ is not informative. On the other hand, the property of being even, though a superset of 2, *is* informative. Consider one definition of evenness:

$$even(x) \stackrel{\text{def}}{=} x \in \mathbb{Z} \wedge x \bmod 2 = 0,$$

where $\mathbb{Z}$ is the set of integers and *mod* is the modulo operator. This adds information about 2 which can be applied to other natural numbers, whereas $x = 2 \vee x = 4$ adds no additional information. (The reader is invited to come up with an analogous definition of shape.)

What are the implications for probabilistic theories of reasoning? One response (in the spirit of Grice, 1989) is to attach a pragmatic theory to the (core) reasoning theory, for instance providing a separate theory of truth conditions and a theory of assertability. Consider the connective ‘but’: its truth conditions match those of ‘and’, but ‘but’ introduces a tension between the conjuncts, so constrains when it is valid to be asserted (see, e.g., McDermott, 1996). This approach of attaching a pragmatic theory to the reasoning theory is taken by mental models theory with its notion of pragmatic modulation (Johnson-Laird and Byrne, 2002), but the exact nature of this modulation is underspecified. Schurz (1991) proposes adding a formal notion of conclusion relevance on top of classical logic, leaving the underlying logic as is. Suppose Γ is a set of premises and φ a conclusion. Then φ is a relevant conclusion from Γ if it follows logically, i.e., $\Gamma \vdash \varphi$, and it is possible to replace any of the predicates in φ with another (of the same arity, etc.) such that φ no longer follows. Otherwise φ is an irrelevant conclusion. Take for instance the inference $x = 2 \vdash x = 2 \vee x = 4$. Since $x = 4$ can be replaced with any other predicate (e.g., $x \neq 4$, $horse(x)$) without affecting validity, the conclusion is irrelevant. Consider instead $x = 2 \vdash even(x)$. Here not all replacements preserve validity, for instance $odd(x)$ would not, so the conclusion i s relevant. One approach we could take, in the spirit of Schurz's suggestion for classical logic, is that uncertain reasoning is modeled by a probability logic plus (at least) a relevance criterion at the meta-level.

Sometimes it is informative to infer a disjunction from one of its disjuncts. Consider the claim,[2] made by a doctor, that if Hans drinks vodka (V) or whiskey (W), then it will be bad for his health (B). Hans drinks whiskey. What follows? It seems likely that people will have little trouble inferring that drinking the whiskey will (if the doctor is to be trusted) be bad for Hans' health. In natural deduction, the proof would go as follows:

$$\dfrac{\dfrac{W}{W \vee V}\,\vee\text{-introduction} \qquad W \vee V \supset B}{B}\,\text{modus ponens}$$

The ‘weakening’ of the known W to $W \vee V$ using $\vee$-introduction is necessary to make the modus ponens go through. It is also arguably informative. The context in which a disjunction is introduced is important for determining its informativeness, as Grice (1989) covers with the side condition that informativeness is relative to the ‘current purposes of the exchange’.

However this is not the only way to model the inference. For instance using the framework of logic programming with negation as failure (suggested by Stenning and van Lambalgen, 2005, for modeling problems where an effect has multiple alternative causes), the premises may be represented as the set:

$$\{W;\ W \wedge \neg ab \rightarrow B;\ V \wedge \neg ab' \rightarrow B;\ \bot \rightarrow ab;\ \bot \rightarrow ab'\}$$

Here the clauses concerning ab (short for ‘abnormal’) allow the modeling of possible exceptions, e.g., that drinking homeopathic quantities of whiskey is unlikely to be bad for Hans. The two terms, $\bot \rightarrow ab$ and $\bot \rightarrow ab'$ indicate that nothing is abnormal ($\bot$ is the always false proposition). Stenning and van Lambalgen (2005) demonstrate how an efficient computation may be used to infer B. This is done without the introduction of a disjunction. Something similar to this can be achieved in a probabilistic framework, with the added benefit of modeling degrees of belief. For instance a conditional independence model may be defined to represent the joint probability distribution of $P(W, V, B)$ such that given information about W or about V affects the probability of B. Again this may be achieved without a disjunction introduction.

Perhaps in some contexts ‘or’ has a different meaning to that in classical logic and probability logic, irrespective of informativeness. The interpretation of disjunction as a modal rather than a truth-functional concept has been proposed (Zimmermann, 2000; Geurts, 2005): formally ‘A or B’ is interpreted as $\diamond A \wedge \diamond B$, where $\diamond\varphi$ denotes that φ is possible. Their approach has not yet been applied to a probabilistic theory, however. The notion of a state of the world being possible is very similar to the notion of a probability being greater than zero. This suggests that the closest approach would involve placing constraints on the disjuncts, e.g., for ‘if A, then A or B’ adding $P(B \mid A) > 0$ to the initial formalization of $P(A \vee B \mid A) = x$, for whatever x is given in the task. ($P(A \mid A) = 1$ already holds, assuming that A is not a logical contradiction.) Again this is a meta-level solution.

Support theory (Tversky and Koehler, 1994) also explains how different ways of referring to the same set can influence the probabilities people infer. A distinction is made in the theory between *packed* descriptions, such as ‘drinks a liquid’ and *unpacked* disjunctions such as ‘drinks coffee, tea, or some other liquid’. The theory addresses issues of how the unpacking process can influence which examples are retrieved from long-term memory, which in turn influences the probability. Originally it was suggested that an unpacked description would have a higher probability than a packed description as it triggers the retrieval of more examples, however sometimes the opposite can be the case and packed descriptions have a higher probability (Sloman et al., 2004). It seems unlikely that in the present experiment people's inferences are influenced by retrieval of shapes, or of even numbers, other than those included in the stimuli. Rather the different expressions of set relations seem to trigger qualitatively different reasoning processes.

Conditional probability responses were also more common for knowledge-based than for stimulus-based dependencies (e.g., ‘If the card shows a 3, then the card shows a square or a circle’). Fugard et al. (in press) hypothesized that reasoning on probabilistic truth table tasks where information about dependences is contained in the stimulus depends on two main steps: the first is obtaining the information from the stimulus about the dependences, which, in the present task, are

[2] Inspired by an example provided by David Over.

obtained from a graphical depictions of cards. The second is inhibiting aspects of this representation of possibilities where the antecedent of the conditional is false. For knowledge-based conditionals, this process should be easier as the information about dependences (e.g., that 2 is an even number) is obtained from long-term memory, rather than from the stimulus, so there is no need to inhibit part of the representation of possibilities.

There is the possibility that some participants use the response 1/2 to mean 'don't know' (see, e.g., Fischhoff and Bruin, 1999) or 'irrelevant'. In our experiment, 1/2 responses were most common for disjunctive conclusions when the dependency was expressed using the stimulus, which, in addition to a zero probability, would indicate that the conclusion was seen as irrelevant. For subset-consequent conditionals, the response according to the conditional event interpretation was also 1/2, so there is the possibility that some of these responses indicated 'don't know'. Future work should investigate whether this is the case, for instance via item selection; providing an additional option asking if an informative probability can be inferred (used previously by Pfeifer and Kleiter, in press); or asking for probability intervals rather than point probabilities (used by Pfeifer and Kleiter, 2005).

Future work should also investigate how people interpret disjunctive conclusions when the premise is uncertain, e.g., given $P(A) = x$, what is the $P(A \vee B)$? For the disjunction introduction, probability logic predicts a conclusion in the interval $[x, 1]$, where x is the probability of one of the disjuncts. Participants who are irrelevance aversive might drop this to the point prediction of 0, or may, following the paradoxes (Pfeifer and Kleiter, in press), opt for a [0,1] conclusion to highlight irrelevance.

To conclude, we have shown that using probability logic, though successful for many aspects of conditional reasoning, does not automatically model all phenomena observed when non-experts reason. These results also pose a problem for other theories of conditionals, for instance the Stalnaker (1968/1991) conditional would predict a high degree of belief for the disjunctive consequent conditionals. Although encoding an 'if' as a conditional event provides a solution to the paradoxes of the material conditional interpretation and allows nonmonotonic reasoning, some additional mechanism for modelling relevance is needed to explain how people interpret disjunctions and other ways of expressing superset relations in natural language. We agree with Stenning and van Lambalgen (2005, p. 929) that 'moving to a probabilistic interpretation of the premisses does not obviate the need for an interpretative process which (a) constructs a model for the premisses, and (b) conducts a computation on the basis of that model'. We have proposed some ways for modeling interpretative processes for uncertain conditionals with consequents referring to a superset of their antecedent.

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